Ways Modality Could Be

~ On the Possibility and Prospects of Higher-Order Modal Logic ~

Jason Zarri

The modal logics of physical and metaphysical necessity are certainly at least as strong as Kρ: If A's truth is determined by the laws of physics/metaphysics, then A is true. But it is not clear that they are any stronger. For example, it is determined by the laws of physics that I do not accelerate through the speed of light. But why should this fact itself be determined by the laws of physics…? Similarly, I am not a frog, and so it is metaphysically possible that I am not a frog. But is that *fact true because of the essence of something…? The essence of possibility?*

— Graham Priest, An Introduction to Non-Classical Logic: From If to Is, 2nd edition, p. 47.

1. *Introduction*

In this paper I introduce the idea of a higher-order modal logic—not a modal logic for higher-order predicate logic, but rather a logic of *higher-order modalities*. "What is a higher-order modality?", you might be wondering. Well, if a first-order modality is a way that some entity could have been—whether it is a mereological atom, or a mereological complex, or the universe as a whole—a higher-order modality is a way that a first-order modality could have been. First-order modality is modeled in terms of a space of possible worlds—a set of worlds structured by an *accessibility relation*, i.e., a relation of relative possibility—each world representing a way that the entire universe could have been. A second-order modality would be modeled in terms of a space of spaces of (first-order) possible worlds, each space representing a way that (first-order) possible worlds could have been. And just as there is a unique *actual world* which represents the way that things actually are, there is a unique *actual space* which represents the way that first-order modality actually is.

One might wonder what the accessibility relation itself is like. Presumably, if it is logical or metaphysical modality that is being dealt with, it is reflexive; but is it also symmetric, or transitive? Especially in the case of metaphysical modality, the answer is not clear. And whichever of these properties it may or may not have, could *that itself* have been different? Could at least *some* rival modal logics represent different ways that first-order modality could have been?

To be clear, the idea behind my proposal is not *just* that some things which are possible or necessary might not have been so at the first order, as determined by the actual accessibility relation, but also that the actual accessibility relation, and hence *the nature or structure of actual modality, could have been different at some higher order of modality.* Even if the accessibility relation is *actually* both symmetric and transitive, perhaps it could (second-order) have been otherwise: There is a (second-order) possible space of worlds in which it is different, where it fails to be symmetric, or transitive. We must, therefore, introduce the notion of a higher-order accessibility relation, one that in this case relates *spaces* of first-order worlds. The question then arises as to whether *that* relation is symmetric, or transitive. We can then consider third-order modalities, spaces of spaces of spaces of possible worlds, where the second-order accessibility relation differs from how *it* actually is. I can see no reason why there should be a limit to this hierarchy of higher-order modalities, any more than I can see a reason why there should be a limit to the hierarchy of higher-order *properties*. There will thus be an infinity of orders, one for each positive integer, and each order will have an accessibility relation of its own. To keep things as clear as possible, a space of first-order points (i.e., of possible worlds) shall be called a *galaxy*, a space of second-order points, a *universe*, and a space of any higher order, a *cosmos*. However, to keep things as *simple* as possible, in what follows I will deal with but a single cosmos at a time, and hence will *not* deal with modalities higher than the third order.

The accessibility relation is not the only thing that might be thought to vary between spaces of worlds: Perhaps the contents of the spaces can vary as well. While I presume that the contents of the worlds themselves remain constant—it makes doubtful sense to suppose that in one space some entity e exists in a world w and in another space e doesn't exist in that same world w—we may suppose that different spaces may differ as to *which worlds they contain*, just as different worlds may differ as to which objects they contain. Thus we might have a higher-order analogue of a variable-domain modal logic. There seem, then, to be three ways in which spaces can differ: First, as to the properties of the accessibility relation; second, as to which worlds the relation relates; and third, as to which worlds or spaces are parts of their domains.

The paper will be structured as follows. In Section 2 I provide some reasons why one might want to pursue this kind of project in the first place. In Section 3 I outline the syntax and semantics of my proposed logic. Section 4 covers semantic tableaux for this system; and after giving the rules for their construction, I construct a few of them myself to establish some logical consequences of the system and give the reader a feel for how it works. In Section 5 I outline a potential application of my framework to the metalogic of modal logics. In Sections 6, 7 and 8 I explore some of its potential philosophical implications for areas besides logic, namely the philosophy of language; metaphysics, including the metaphysics of modality, the philosophy of time, and laws of nature; and finally the philosophy of religion, before concluding the paper in Section 9.

2. *Motivation*

Why should we adopt a framework such as the one I have just described? To motivate it, consider the fact that people have mutually conflicting intuitions about what the space of all (first-order) possible worlds is like. For example, does God exist in all, none, or only some worlds? Each of these positions, if true, is necessarily true, and if false, is necessarily false. God either exists in all, none, or only some worlds, and on the standard view the actual space of worlds could not have been different, because it is the only space of possible worlds that there is.

On the face of it, this is problematic for the view that conceivability implies possibility: Each of these positions has been believed, and by very able philosophers at that. What is believed is conceivable in some sense; otherwise, such "beliefs" would have no content. So each position is conceivable, but only one is possible. No matter which of them holds, conceivability doesn't imply possibility.

But maybe that's not *quite* true. Perhaps, though only one of these positions is actually true, and hence first-order possible, each is *second-order possible*. So maybe conceivability *does* imply possibility—at some order or other. Related considerations might apply to semantic content and possibility: If we can coherently mean something, it can be the case—at some order or other. If one is already committed to the idea that conceivability implies possibility, one has a reason to be interested in my project.

Or consider Kripke's doctrine of the essentiality of origins. According to him, for example, your parents are essential to your existence; any metaphysically possible world where you exist is a world in which you have the same parents that you do in *this* word, the actual world (*Naming and Necessity*, pp. 112-3). Even if we assume that this is so as a matter of first-order metaphysical necessity, it seems to be higher-order contingent: Surely your having the parents you do is not a matter of *logic*, but in that case it is logically possible for it to be otherwise. Furthermore, it doesn't seem to be a matter of logic that it is *(first-order) metaphysically necessary* that you have the parents you do, and hence it seems *logically possible* that it is *not* (first-order) metaphysically necessary that you have them as your parents. Perhaps, then, even if every metaphysically possible world in the actual space of worlds is one in which you have the parents that you actually do, the actual galaxy accesses a possible galaxy which contains metaphysically possible worlds where you have different parents. My framework, then, can explain how claims that hold at every first-order metaphysically possible world could nevertheless be higher-order contingent.

Finally, and perhaps most importantly, consider the fact that there are many modal logics, which are mutually inconsistent, in the sense that two or more such logics cannot correctly describe the structure of the same space of worlds. These logics all seem to be perfectly good, and perfectly *intelligible*, as systems of modal logic. Even if I suppose that S5 (i.e. Kρστ), for instance, correctly represents the actual structure of metaphysical modality, I can wonder how things would have differed if the accessibility relation had lacked certain properties; and another system, say S4 (i.e. Kρτ), can tell me exactly what characteristically *modal* arguments would have been valid if that had been so—or so it seems to me. S4 certainly constitutes a *representation* of the structure—or better, *a* structure—of metaphysical modality, even if it is *actually* a necessarily false one. *Qua* representation, it seems to me to have much the same status as our more customary representations of how certain things about our world would have turned out if certain other things about our world had been different. In much the same way that we take these representations to depict ways things could have been, why couldn't we take S4 and other modal logics to depict ways *modality* could have been?

I should make it clear that I do not expect the kind of framework I propose to *settle* the issue of how modality at any order actually is—no more than I expect ordinary first-order modal logic to *settle* (aside from first-order necessary truths) what is actually the case. What goes for the actual world goes for the actual space of worlds, and for all higher-order spaces of spaces. What I do hope for is that it will, if it proves to be coherent, help to clarify the terms of the debate about the way modality is—to help us to state the issues, and to see their interrelations, as clearly as we can.

3. *Setting up the System: Syntax and Semantics*

In this section I shall describe the syntax and semantics of my proposed system, which I shall call 'HOML', an acronym for 'Higher-Order Modal Logic'.

3.1. *Syntax*

First, we have the *syntax*:

1) I shall use $\Box_n$ and $\Diamond_n$, respectively, for necessity and possibility operators of the order n, for positive integers 1, 2 and 3.

2) I shall use $\Box_n^m$ and $\Diamond_n^m$, respectively, for an iteration of m necessity or possibility operators of the order n.

3) For atomic sentences, we will use capital letters from the entire alphabet, with numerical subscripts appended to them if necessary.

4) As for connectives, we will use the symbols ¬ , ∧ , ∨ , ⊃ , and ≡ , respectively, for negation, conjunction, disjunction, material implication, and material equivalence.

5) For brackets one can use parentheses, (,) ; square brackets, [,] ; or curly brackets, {, }. It makes no difference which brackets are used where.

6) I will use 1 and 0 as truth values, 1 for *true* and 0 for *false*.

7) Finally, the usual recursive clauses for constructing *well-formed formulas*—*wffs*, pronounced "woofs," for short—from atomic sentences will be adopted. All atomic sentences of HOML are wffs, and where p and q are arbitrary sentences of HOML:

1. If p is a wff, so are ¬ p, $\Box_1$ p, $\Box_2$ p, $\Box_3$ p, $\Diamond_1$ p, $\Diamond_2$ p, and $\Diamond_3$ p.
2. If p and q are wffs, so is (p ∧ q).
3. If p and q are wffs, so is (p ∨ q).
4. If p and q are wffs, so is (p ⊃ q).
5. If p and q are wffs, so is (p ≡ q).
6. Nothing else is a wff of HOML.

3.2. *Semantics*

1) Any set of worlds that is structured by an accessibility relation, or a higher-order counterpart, is a *space*, and its members are *points*. As above, a space of first-order points (i.e., of possible worlds) shall be called a *galaxy*, a space of second-order points, a *universe*, and a space of any higher order, a *cosmos*. Points of these orders shall be represented by expressions of the forms w^x, g^y, and u^z, as mnemonics, respectively, for 'world x', 'galaxy y', and 'universe z'.

2) Every space has within it an accessibility relation holding between its points, and as every non-base point is itself a space, it will have an accessibility relation holding between *its* points. Thus, when considering a given universe of galaxies, one must take into account the fact that there will be a different accessibility

relation for each galaxy, and that the properties of these relations may differ. So in this universe, there will be an accessibility relation of the second order which holds between its galaxies, and many accessibility relations of the first order holding between the possible worlds within the galaxies. If the universe we are considering is u^1, I will call the accessibility relation that holds between its galaxies $R^1{}_g$: The superscripted ‘1’ means that the relation holds within universe u^1, and the subscripted ‘g’ means that it holds between galaxies. If, within u^1, we are considering the galaxy g^3, I will similarly call the accessibility relation that holds between its worlds $R^3{}_w$. Here the ‘3’ indicates that we are dealing with the relation that holds within galaxy g^3, and the ‘w’ indicates that it holds between possible worlds.

3) Model structures: A *cosmos* is a 6-tuple <**W**, **G**, **U**, **RW**, **RG**, **RU**> where:

W is a non-empty set, its members are possible worlds,
G is a non-empty set of subsets of **W**, its members are galaxies,
U is a non-empty set of subsets of **G**, its members are universes,
RW = {$R^y{}_w$: g^y in **G**} is a set of access relations, one for each galaxy, and defined on that galaxy, holding between possible worlds.
RG = {$R^z{}_g$: u^z in **U**} is a set of access relations, one for each universe, and defined on that universe, holding between galaxies.
RU = {R_u} (no superscript necessary in this case) is the unit set of the access relation defined on the cosmos, holding between universes.

An evaluation point is a triple <$w^xg^yu^z$>, with a world w^x in **W**, a galaxy g^y in **G**, and a universe u^z in **U** and such that w^x is in g^y and g^y is in u^z.

4) Truth-values are assigned to sentences relative to these points, like so:
For an atomic sentence, the truth-value depends only on w (it is the same for <w^x g^y u^z> and <w^x g^i u^k>). Writing the evaluation function, which assigns the

semantic values 1 and 0 to sentences, as v (p) @ $<w^x g^y u^z>$, meaning the value of p at the triple $<w^x g^y u^z>$, I define:

1) $v(\neg p)$ @ $<w^x g^y u^z> = 1$ iff $v(p)$ @ $<w^x g^y u^z> = 0$
2) $v(p \wedge q)$ @ $<w^x g^y u^z> = 1$ iff $v(p)$ @ $<w^x g^y u^z> = 1$ and $v(q)$ @ $<w^x g^y u^z> = 1$
3) $v(p \vee q)$ @ $<w^x g^y u^z> = 1$ iff $v(p)$ @ $<w^x g^y u^z> = 1$ or $v(q)$ @ $<w^x g^y u^z> = 1$
4) $v(p \supset q)$ @ $<w^x g^y u^z> = v(\neg(p \wedge \neg q))$ @ $<w^x g^y u^z>$
5) $v(p \equiv q)$ @ $<w^x g^y u^z> = v((p \supset q) \wedge (q \supset p))$ @ $<w^x g^y u^z>$
6) $v(\Box_1 (p))$ @ $<w^x g^y u^z> = 1$ iff, for all worlds w' in g^y in u^z that w^x accesses, $v(p)$ @ $<w' g^y u^z> = 1$.
7) $v(\Box_2\, p)$ @ $<w^x g^y u^z> = 1$ iff, for all worlds w' in all galaxies g' in u^z that g^y accesses, $v(p)$ @ $<w' g' u^z> = 1$.
8) $v(\Box_3\, p)$ @ $<w^x g^y u^z> = 1$ iff, for all worlds w' in all galaxies g' in all universes u' that u^z accesses, $v(p)$ @ $<w' g' u'> = 1$.
9) $v(\Diamond_1\, p)$ @ $<w^x g^y u^z> = 1$ iff, for some world w' in g^y in u^z that w^x accesses, $v(p)$ @ $<w' g^y u^z> = 1$.
10) $v(\Diamond_2\, p)$ @ $<w^x g^y u^z> = 1$ iff, for some world w' in some galaxy g' in u^z that g^y accesses, $v(p)$ @ $<w' g' u^z> = 1$.
11) $v(\Diamond_3\, p)$ @ $<w^x g^y u^z> = 1$ iff, for some world w' in some galaxy g' in some universe u' that u^z accesses, $v(p)$ @ $<w' g' u'> = 1$.

5) Definition of *satisfaction*: 1 is the sole designated value in HOML. A sentence p is *satisfied* with respect to a point of evaluation $<w^x g^y u^z>$ iff it is assigned a designated value at $<w^x g^y u^z>$. A set **X** of sentences is *satisfied* with respect to a point of evaluation iff every member of **X** is satisfied at that point.

6) *Logical consequence.* We say that a sentence p is a *logical consequence* of a set **X** of sentences, which we write as $\mathbf{X} \models p$, iff in every model, i.e., in every cosmos **C** where **X** is satisfied with respect to a point of evaluation, p is also satisfied at that point. If there is

some cosmos **C** where **X** is satisfied with respect to a point of evaluation and p is not, we say that **C** is a *countermodel* to **X** $\models$ p. Furthermore, if **X** is the empty set and **X** $\models$ p, we say that p is a *logical truth* or *logically valid sentence*, and may simply write $\models$ p.

7) *Restricted logical consequence.* We say that a sentence p is a *restricted logical consequence* of a set **X** of sentences iff in every cosmos **C**, *such that C has certain desired properties that are not guaranteed by the semantics alone*, and where **X** is satisfied with respect to a point of evaluation, p is also satisfied at that point. The desired properties are properties of the accessibility relations involved in the cosmos, namely being reflexive (abbreviated as 'REF.'), symmetric (SYMM.), transitive (TRANS.), or serial (SER.). If p is a restricted logical consequence of **X** when we assume that the accessibility relations have one or more of these properties, we write **X** $\models$ p (or $\models$ p if p is a *restricted theorem*, i.e., a consequence of the empty set if certain assumptions about accessibility relations are made) followed the names of the relation(s) and the abbreviation(s) of the assumed property(ies) enclosed in square brackets, followed by a comma, and then followed by any other relation(s) and the abbreviation(s) of the assumed property(ies) enclosed in square brackets, and so on, until they have all been mentioned. For example, if p is a logical consequence of **X** when we assume that R^y_w is symmetric, we write **X** $\models$ p [R^y_w SYMM]. And if p is a logical consequence of X when we assume that R^z_g is both reflexive and transitive, we write **X** $\models$ p [R^z_g REF. TRANS.]. As a final example, if p is a logical consequence of **X** when we assume that R^z_g is both reflexive, symmetric and transitive and that R_u is reflexive, we write **X** $\models$ p [R^z_g REF. SYMM. TRANS.], [R_u REF.].

4. *Semantic Tableaux*

In this section I first describe the tableaux rules associated with HOML, and then construct some actual tableaux to establish some consequences of the system.

4.1. *Tableaux Rules*

I will use semantic tableaux to establish *derivability*. We say that a sentence p is *derivable from* a set **X** of sentences, which we write as **X** ├ p, iff every tableau containing every member of **X** at the root node, followed by the negation of p at the root node is *closed*. A tableau is closed iff every one of its branches is *closed*, and a branch is *closed* iff for some sentence q that branch contains both q and the negation of q. Furthermore, every node of a tableau will be followed by a comma and the name of the evaluation point at which it holds. Tableaux for the truth-functional connectives will have the following forms:

$\neg\neg p, <w^x g^y u^z>$
|
$p, <w^x g^y u^z>$

$p \wedge q, <w^x g^y u^z>$
|
$p, <w^x g^y u^z>$
$q, <w^x g^y u^z>$

$\neg(p \wedge q), <w^x g^y u^z>$
/ \
$\neg(p), <w^x g^y u^z>$ $\quad$ $\neg(q), <w^x g^y u^z>$

$p \vee q, <w^x g^y u^z>$
/ \
$p, <w^x g^y u^z>$ $\quad$ $q, <w^x g^y u^z>$

$\neg(p \vee q), <w^x g^y u^z>$
|
$\neg(p), <w^x g^y u^z>$
$\neg(q), <w^x g^y u^z>$

$p \supset q, <w^x g^y u^z>$
/ \
$\neg(p), <w^x g^y u^z>$ $\quad$ $q, <w^x g^y u^z>$

$\neg(p \supset q), <w^x g^y u^z>$
|
$p, <w^x g^y u^z>$
$\neg(q), <w^x g^y u^z>$

$p \equiv q, <w^x g^y u^z>$
/ \
$p, <w^x g^y u^z>$ $\quad$ $\neg(p), <w^x g^y u^z>$
$q, <w^x g^y u^z>$ $\quad$ $\neg(q), <w^x g^y u^z>$

$\neg(p \equiv q), <w^x g^y u^z>$
/ \
$p, <w^x g^y u^z>$ $\quad$ $\neg(p), <w^x g^y u^z>$
$\neg(q), <w^x g^y u^z>$ $\quad$ $q, <w^x g^y u^z>$

Tableaux for modal operators will have the following forms:

$$\begin{array}{c} \Diamond_1 p, \langle w^x g^y u^z \rangle \\ | \\ w^x\ R^y{}_w\ w^i \\ | \\ p, \langle w^i g^y u^z \rangle \end{array} \qquad \begin{array}{c} \Box_1 p, \langle w^x g^y u^z \rangle \\ | \\ w^x\ R^y{}_w\ w^i \\ | \\ p, \langle w^i g^y u^z \rangle \end{array}$$

(where w^i is new to the branch), (for all worlds that are members of g^y)

$$\begin{array}{c} \Diamond_2 p, \langle w^x g^y u^z \rangle \\ | \\ g^y\ R^z{}_g\ g^j \\ | \\ p, \langle w^i g^j u^z \rangle \end{array} \qquad \begin{array}{c} \Box_2 p, \langle w^x g^y u^z \rangle \\ | \\ g^y\ R^z{}_g\ g^j \\ | \\ p, \langle w^i g^j u^z \rangle \end{array}$$

(where g^j is new to the branch and w^i is a member of it), (for all worlds of all galaxies which g^y accesses that are members of u^z)

$$\begin{array}{c} \Diamond_3 p, \langle w^x g^y u^z \rangle \\ | \\ u^z\ R_u\ u^k \\ | \\ p, \langle w^i g^j u^k \rangle \end{array} \qquad \begin{array}{c} \Box_3 p, \langle w^x g^y u^z \rangle \\ | \\ u^z\ R_u\ u^k \\ | \\ p, \langle w^i g^j u^k \rangle \end{array}$$

(where u^k is new to the branch, g^j is a member of u^k, and w^i is a member of g^j), (for all worlds of all galaxies of all universes which u^z accesses)

In addition, where n can be either 1, 2 or 3, we have:

$$\begin{array}{c} \neg \Diamond_n p, \langle w^x g^y u^z \rangle \\ | \\ \Box_n \neg p, \langle w^x g^y u^z \rangle \end{array} \qquad \begin{array}{c} \neg \Box_n p\ \langle w^x g^y u^z \rangle \\ | \\ \Diamond_n \neg p, \langle w^x g^y u^z \rangle \end{array}$$

4.2. *Some Semantic Tableaux*

I will now construct a few schematic semantic tableaux in order to establish some consequences of the system. First, I will establish that $\Box_2\, p \vdash \Box_1\, p$ [R^1_g REF.] :

$$
\begin{array}{c}
\Box_2\, p, \langle w^1 g^1 u^1 \rangle \\
\neg \Box_1\, p, \langle w^1 g^1 u^1 \rangle \\
| \\
\Diamond_1 \neg p, \langle w^1 g^1 u^1 \rangle \\
| \\
w^1\ R^1_w\ w^2 \\
| \\
\neg p, \langle w^2 g^1 u^1 \rangle \\
| \\
g^1\ R^1_g\ g^1 \text{ [REF.]} \\
| \\
p, \langle w^2 g^1 u^1 \rangle \\
\mathbf{x}
\end{array}
$$

Thus, if something is second-order necessary it is also first-order necessary. We can say that necessity is *hereditary downwards*.

Second, I will establish that $\Diamond_1\, p \vdash \Diamond_2\, p$ [R^1_g REF.] :

$$
\begin{array}{c}
\Diamond_1\, p, \langle w^1 g^1 u^1 \rangle \\
\neg \Diamond_2\, p, \langle w^1 g^1 u^1 \rangle \\
| \\
\Box_2 \neg p, \langle w^1 g^1 u^1 \rangle \\
| \\
w^1\ R^1_w\ w^2 \\
| \\
p, \langle w^2 g^1 u^1 \rangle \\
| \\
g^1\ R^1_g\ g^1 \text{ [REF.]} \\
| \\
\neg p, \langle w^2 g^1 u^1 \rangle \\
\mathbf{x}
\end{array}
$$

Thus if something is first-order possible it is also second order possible. So possibility is *hereditary upwards*.

Third, I will establish that $\Box_2 p \vdash \Box_1\Box_2 p$:

$$\Box_2\, p, <w^1g^1u^1>$$
$$\neg\, \Box_1 \Box_2\, p, <w^1g^1u^1>$$
$$|$$
$$\Diamond_1 \neg\, \Box_2\, p, <w^1g^1u^1>$$
$$|$$
$$w^1\ R^1{}_w\, w^2$$
$$|$$
$$\neg\, \Box_2\, p, <w^2g^1u^1>$$
$$|$$
$$\Diamond_2 \neg\, p, <w^2g^1u^1>$$
$$|$$
$$g^1\ R^1{}_g\, g^2$$
$$|$$
$$\neg\, p, <w^3g^2u^1>$$
$$|$$
$$p, <w^3g^2u^1>$$ (from the first premise and "$g^1\ R^1{}_g\, g^2$")

x

Thus, if something is second-order necessary, it is first order necessary that it is second order necessary.

I believe these three tableaux will suffice to give the reader an idea of how to construct tableaux for HOML, and an appreciation of some of its general features.

5. *An Application to Modal Metalogic: Kripke Cosmoi*

In this section I will single out a special subset of HOML's models, or cosmoi. I shall call these *Kripke cosmoi* because they contain components corresponding to the logic K—whose name derives from the work of Saul Kripke—and its extensions.

Kripke cosmoi, or ***K*** cosmoi for short, each contain exactly one universe, which contains indefinitely many galaxies, each galaxy containing an arbitrary number (greater than zero) of worlds. To distinguish these special models from the rest, I will use a slightly modified syntax. Instead of using a ‘g’ to denote a galaxy, I shall use a ‘k’, for ‘Kripke’. Thus galaxy k (no superscript necessary) corresponds to the logic K, k^{ρ} to the logic Kρ …, and $k^{\rho\sigma\tau}$ to the logic Kρστ. Correspondingly, we shall write ‘R_k’ for the access relation that holds between the galaxies. (Since there is only one universe in each **K** cosmos, there is no need to superscript a numeral either to terms denoting it or the access relation defined on it.) Points will then be of the form $<w^x k^y u>$. Furthermore, we shall change the notation for the second and third-order modal operators, and accordingly drop the subscripts for the first-order ones. Instead of $\Diamond_2$, we have <E>, and instead of $\Box_2$, we have [E]. And instead of $\Diamond_3$, we have <K>, and instead of $\Box_3$, we have [K]. The reason for this change of notation is as follows: In each **K** cosmos the universe u accesses itself, so R_u is reflexive; and since there are no other universes it is trivially symmetric and transitive. Thus ⌜<K> p⌝ has the effect of saying that p holds in at least one galaxy, and hence, in K or in at least one of its extensions; while ⌜[K] p⌝ has the effect of saying that p holds in K and in all of its extensions. Also, in each **K** model, R_k is irreflexive, asymmetric, and intransitive; and for each galaxy, it accesses another just in case the logic corresponding to it is (properly) extended by the logic corresponding to the other. Since Kρ extends K, we have $k \; R_k \; k^{\rho}$; since Kρστ extends Kρτ, we have $k^{\rho\tau} \; R_k \; k^{\rho\sigma\tau}$; and so on. Thus ⌜<E> p⌝ has the effect of saying that p holds in at least one galaxy that (properly) “extends” a given galaxy, and ⌜[E] p⌝ has the effect of saying that p holds in all galaxies that (properly) extend a given galaxy. So R_k mimics the relation ‘being (properly) extended by’ which holds between K and its extensions. Finally, as the reader may have guessed, the access relation that holds between the worlds of a given galaxy corresponds to that of the logic which that galaxy represents. So, the relation R_w (with no superscript, since it is the relation that holds within the galaxy k) is never assumed to be reflexive, symmetric or transitive; the relation $R^{\rho\sigma\tau}{}_w$ is assumed to be reflexive, symmetric and transitive; and correspondingly for the access relations of the other galaxies. When logical consequence is restricted to **K** cosmoi, we shall write **X** ╞ p [**K**] for ⌜q is a semantic **K**-consequence of **X**⌝ and **X** ├ p [**K**] for ⌜q is a derivable **K**-consequence of **X**⌝.

The reader may now be able to see why **K** cosmoi are special: they allow one to reason about the properties of and relations between the logic K and its extensions in a relatively simple and straightforward manner. I have no doubt that similar methods could be applied to model the properties and relations that apply to modal logics of other kinds. So, one potential benefit of HOML is that it gives us an easy way to investigate, at the level of an object language, the metalogical properties and relations that apply to modal logics by simulating the relation whereby one such logic (properly) extends another.

6. *Implications for the Philosophy of Language*

I believe that my account has consequences for the philosophy of language; specifically, for a variety of counterfactual conditionals known as *counterpossibles*. Counterpossibles are counterfactuals that have impossible antecedents. As an example, let us take once more Kripke's thesis of the essentiality of origins. Let's suppose that Kripke is right, and that one essentially has the parents one actually has. Let us also suppose that the framework I have proposed is correct, and additionally that it is second-order possible that one has different parents from one's actual ones. One can then make good sense of the counterpossible conditional, “If the thesis of the essentiality of origins were false, Quine could have had Carnap for a father." On the standard semantics, this is true, but vacuously so. For granting the essentiality of origins the antecedent is impossible, and on the standard semantics all counterfactuals with impossible antecedents are true, including the conditionals "If the thesis of the essentiality of origins were false, Quine *could not* have had Carnap for a father," and "If the thesis of the essentiality of origins were false, Quine would have been a fried egg." On my approach, however, one could construct a semantics in which, if a sentence is not true at any world in the actual galaxy, one can look to worlds in other possible galaxies. The first conditional can then come out as non-vacuously true, and the second and third as non-vacuously false. Moreover, one need not invoke "intrinsically impossible" worlds—worlds which are impossible *full stop,* in and of themselves—but only worlds which are impossible *with respect to* worlds in the actual galaxy. (One could, of course, modify my framework to include intrinsically impossible worlds, perhaps to accommodate counterpossibles whose antecedents contravene logical laws, but one would then have to make non-trivial changes to HOML.)

7. *Implications for Metaphysics*

I believe that my account is relevant to three metaphysical issues: the metaphysics of modality, the philosophy of time, and laws of nature.

7.1. *The Metaphysics of Modality*

"The world is the totality of facts, not of things" (*Tractatus Logico-Philosophicus* 1.11). So said Wittgenstein in the opening lines of the *Tractatus*. But is that true?

If we "modalize" and "pluralize" Wittgenstein's account, so that it identifies possible worlds with totalities of what *would be* facts if they *were* actualized, I think the answer must be "no" if my framework is to be accepted with the interpretation that I have given it. This is because evaluation points are ordered n-tuples rather than worlds. Consider two points (using ordered triples, as I did in HOML, for the sake of simplicity): $<w^1g^1u^1>$ and $<w^1g^2u^1>$. Now, since w^1 occurs in both these points, they will agree on the truth values of all their atomic sentences, and hence on the truth values of all their logical consequences, including all truth functional compounds which are built up ultimately from these atomic sentences. However, they might not agree on the truth values of their modal sentences, and hence on any of *their* logical consequences. Thus, on my view we cannot speak of the totality of what *would be facts*—of what *would be the case*—if a given possible world were actualized, for the truth values of the totality of its atomic sentences and their logical consequences underdetermines the truth values of modal sentences and *their* logical consequences, and hence underdetermines the truth values of the purported totality of sentences that would be true if that world were actualized. If we *did* identify a possible world with the totality of what would have been true if that world were actualized, then, we would have to conclude the truth values of modal sentences which hold concerning a world could not have differed from what they actually are, and that would undermine the interpretation that I have given of HOML.

If, however, we identify a possible world with the totality of what would have been true *no matter which of the points that contain it had been actualized*, there is no problem. In that case possible worlds cannot be thought of as totalities of everything that would have been true if they had been actualized, since *there are no such totalities*. But they *can* be thought of as being totalities of *certain things* that would be true if they had been actualized—namely, certain atomic sentences and their logical consequences. Indeed, they are even maximal in a sense: For every atomic sentence, a world includes either that sentence or its negation. If we call atomic sentences and their negations *literals*, we could think of possible worlds as maximal sets of literals (and their logical consequences). They might not be quite as "big" as analytic-philosophical commonsense would have them be—since they aren't maximal sets of sentences *full stop*—but they seem big enough to serve as possible worlds, or at least as useful replacements for those not-entirely-unambiguously-understood entities. So my framework has some consequences for the metaphysics of modality, but I doubt that they are very momentous.

However, since two different points which contain the same possible world may disagree on modal sentences, but will have to agree on what exists (since they both contain the same possible world) we must hold that either truthmaker maximalism, or truthmaker necessetarianism, or what has been called 'thingism' is false: The totality of what is true is not fixed by the totality of what exists. (The status of the thesis that truth supervenes on being is not so clear. For the record, my own inclination is to reject necessetarianism and/or thingism.) This is a significant consequence, but I must leave the examination of what exactly its significance is for another occasion.

Those who believe in modality are divided into two camps. David Lewis labeled one the 'modal realist' camp, and the other the 'modal ersatzist' camp (*On the Plurality of Worlds*, Chapter 3). Those who are not Lewisian modal realists may take possible worlds be various abstract entities: properties, or sets of properties, or maximal sets of propositions, or Plantingan maximal states of affairs, or perhaps something different still. Whichever of these views one takes, I think adopting my framework simply gives one “more of the same”: One can take evaluation points to be properties, or sets of properties, or maximal sets of propositions, or Plantingan maximal states of affairs … and one can identify possible worlds (and galaxies, and universes…) with certain parts or subsets or other components of these. Something similar, I take it, goes for instrumentalism: If one can be an instrumentalist about ordinary possible worlds semantics, one can be an instrumentalist about HOML as well. If I'm right about all this, the main difference between my view and first-order views is that mine posits more structure in evaluation points than theirs does, in the form of accessibility relations between parts or subsets or other components of these entities. Thus I think the higher-order versions of each of the above views will have the same sorts of pros and cons as the first-order views of which they are extensions, and hence that adopting my framework should leave the debate over the metaphysics of modality much as it was before.

I think modal realism is an exception to that conclusion, because the notion of a higher-order accessibility relation doesn't make much sense on that view, but then first-order accessibility relations don't make much sense on that view either. On the Lewisian picture, possible worlds are maximal spatio-temporally connected concrete entities; and since his is a *reductive* account of modality, there's not much, if any, content to the 'possible' in 'possible world': Their “possibility” amounts to their *existence,* and perhaps also to the fact that it is analytically true that nothing existent can contain the Russell set, married bachelors, round squares, and the like. A primitive notion of relative possibility could then make sense only if a primitive notion of relative existence made sense, which (I think) it does not. And if the notion of relative possibility were not primitive, on the Lewisian picture, such relations would have to supervene on the non-modal properties and relations of possible worlds. It would then make no sense to suppose that such relations could be different from how they actually are. My framework, then, is incompatible with modal realism, if modal realism is taken to be a reductive account of modality. There is, however, no incompatibility between my framework and the existence of multiple maximal spatio-temporally connected concrete entities, only with the idea that modality is correctly analyzed in terms of them.

7.2. *The Philosophy of Time*

There is no reason why a framework like mine must be limited to logical and metaphysical modality: One can modify HOML to get higher-order tense logic(s). "Possible worlds" become moments of time, "possible galaxies" become intervals of time, or even entire timelines, and can themselves be considered "second-order moments" of "second order time." The second-order relation of relative possibility is re-interpreted as a *second-order earlier than* or a *second-order later than* relation. The idea here is that second-order moments represent different ways that first-order moments can be related by the first-order earlier than or later than relations, and that which of those first-order relations obtains could change over the course of second-order time. Second-order moments could also, of course, differ as to which first-order moments they contain, and hence which first-order moments are part of the first-order timeline could also change over the course of second-order time.

Just as, on first-order views, time can be depicted as a one-dimensional line, on this view time could be depicted as a multi-dimensional space. Since HOML's evaluation points are ordered triples its temporal counterpart will represent time in a way that could be visualized as a three-dimensional space. The first dimension represents the arrangement and contents of first-order moments of time: How objects come to be, cease to be, alter, or remain the same. The second dimension represents the arrangement and contents of second-order moments of time: How intervals of time, which consist of first-order moments, come to be, cease to be, alter, or remain the same. Finally, the third dimension represents the arrangement and contents of third-order moments of time: How "temporal planes" of time, which consist of intervals, come to be, cease to be, alter, or remain the same. If one is a B-theorist, one can take this talk of dimensions literally: If time is fundamentally similar to space, why could there not be multiple temporal dimensions, just as there are multiple spatial dimensions?

B-theorists can take two stances regarding the difference between time and space. First, they could say that there is no intrinsic difference; we simply construe one dimension of a manifold as temporal because, e.g., it is the dimension in which entropy increases in one direction. On this view, I cannot see why there couldn't be additional dimensions in which entropy—in this case, a measure of the disorder of an entire *interval* of time, or of an entire *"temporal plane"*—increases in one direction. Second, they could say that space and time, though highly similar, are qualitatively distinct. On this view matters are less clear, but it would strike me as curious if there could be many dimensions of space, but not of time. I would regard arguments to that effect with as much suspicion as I would arguments to the effect that space must be Euclidean or no more than three dimensional. Even if time must be one-dimensional, the discovery of such things as non-Euclidean geometries and their physical application should caution us not to regard time's mono-dimensionality as something that can be settled a priori.

What, though, of A-theorists? On their view time is not like space, and tenses or tensed properties must be taken as primitive or irreducible. Here matters are still less clear, so I will simply say that if one is free to posit first-order tenses or tensed properties as primitive or irreducible, why shouldn't one be free to posit higher-order tenses or tensed properties as primitive or irreducible? At this point I suspect that arguments would rest on a clash of intuitions, with no effective means of deciding between them. It would be better, I think, to simply explore higher-order tense logic in order to see what benefits, if any, we can derive from it; and then, if we find them to be sufficiently great, we can accept it as a good *working hypothesis*. If it escapes attempts to prove it metaphysically incoherent, well and good. If not, we could still regard it as a useful fiction.

I will now outline the syntax and semantics of such a system, which I call 'HOTL', an acronym for 'Higher-Order Tense Logic'. In 7.2.1., I give the syntax of HOTL; in 7.2.2., its semantics; in 7.2.3., its tableaux rules, and in 7.2.4., I construct some tableaux. Finally, in 7.2.5. an interesting application of HOTL is briefly sketched.

7.2.1. *Syntax of HOTL*

The syntax is the same as in HOML, except for the modal operators. In their place HOTL has tense operators:

> If p is a wff of HOTL, so are $[P]_1 p$, $[P]_2 p$, $[P]_3 p$, $<P>_1 p$, $<P>_2 p$, $<P>_3 p$, $[F]_1 p$, $[F]_2 p$, $[F]_3 p$, $<F>_1 p$, $<F>_2 p$, and $<F>_3 p$.

Here $[P]_n p$ means that it was always the case that p at order n; $<P>_n p$, that it was at some time the case that p at order n; $[F]_n p$, that it will always be the case that p at order n; and $<F>_n p$, that it will at some time be the case that p at order n.

7.2.2 *Semantics of HOTL*

1) A space of first-order points, i.e., of times, is called an *interval*; a space of second-order points, a *span*; and a space of any higher order, an *eon*. Since we are only dealing with three orders of time, we will use but a single eon, considered as a model structure.

2) An eon is a 6-tuple <**T**, **I**, **S**, **RT**, **RI**, **RS**> where:

T is a non-empty set, its members are times,
I is a non-empty set of subsets of **T**, its members are intervals,
S is a non-empty set of subsets of **I**, its members are spans,
RT = {R^y_t: i^y in **I**} is a set of access relations, one for each interval, and defined on that interval, holding between times.
RI = {R^z_i: s^z in **S**} is a set of access relations, one for each span, and defined on that span, holding between intervals.
RS = {R_s} (no superscript necessary in this case) is the unit set of the access relation defined on the eon, holding between spans.

An evaluation point is a triple $<t^x i^y s^z>$, with a time t^x in **T**, an interval i^y in **I**, and a span s^z in **S** and such that t^x is in i^y and i^y is in s^z.

3) Truth values are assigned to points in the same way as in HOML, except for the modal operators. The truth conditions for the tense operators are:

1) $v([P]_1\, p)$ @ $<t^x i^y s^z>$ = 1 iff, for all times t' in i^y that access t^x, $v(p)$ @ $<t' i^y s^z>$ = 1.

2) $v([P]_2\, p)$ @ $<t^x i^y s^z> = 1$ iff, for all times t’ in all intervals i’ in s^z that access i^y, $v(p)$ @ $<t'i's^z> = 1$.

3) $v([P]_3\, p)$ @ $<t^x i^y s^z> = 1$ iff, for all times t’ in all intervals i’ in all spans s’ that access s^z, $v(p)$ @ $<t'i's'> = 1$.

4) $v(<P>_1\, p)$ @ $<t^x i^y s^z> = 1$ iff, for some time t’ in i^y that accesses t^x, $v(p)$ @ $<t'i^y s^z> = 1$.

5) $v(<P>_2\, p)$ @ $<t^x i^y s^z> = 1$ iff, for some time t’ in some interval i’ in s^z that accesses i^y, $v(p)$ @ $<t'i's^z> = 1$.

6) $v(<P>_3\, p)$ @ $<t^x i^y s^z> = 1$ iff, for some time t’ in some interval i’ in some span s’ that accesses s^z, $v(p)$ @ $<t'i's'> = 1$.

7) $v([F]_1\, p)$ @ $<t^x i^y s^z> = 1$ iff, for all times t’ in i^y that t^x accesses, $v(p)$ @ $<t'i^y s^z> = 1$.

8) $v([F]_2\, p)$ @ $<t^x i^y s^z> = 1$ iff, for all times t’ in all intervals i’ in s^z that i^y accesses, $v(p)$ @ $<t'i's^z> = 1$.

9) $v([F]_3\, p)$ @ $<t^x i^y s^z> = 1$ iff, for all times t’ in all intervals i’ in all spans s’ that s^z accesses, $v(p)$ @ $<t'i's'> = 1$.

10) $v(<F>_1\, p)$ @ $<t^x i^y s^z> = 1$ iff, for some time t’ in i^y that t^x accesses, $v(p)$ @ $<t'i^y s^z> = 1$.

11) $v(<F>_2\, p)$ @ $<t^x i^y s^z> = 1$ iff, for some time t’ in some interval i’ in s^z that i^y accesses, $v(p)$ @ $<t'i's^z> = 1$.

12) $v(<F>_3\, p)$ @ $<t^x i^y s^z> = 1$ iff, for some time t’ in some interval i’ in some span s’ that s^z accesses, $v(p)$ @ $<t'i's'> = 1$.

4) The definitions of satisfaction, validity, and restricted validity given for HOML can be modified to obtain ones for HOTL in an obvious way.

7.2.3. *Tableaux Rules for HOTL*

The tableaux rules for the truth functional connectives are the same as those for HOML, except of course that the points are now triples of the form $<t^x i^y s^z>$. The tableaux rules for the tense operators are:

$$\begin{array}{c} <P>_1\, p,\ <t^x i^y s^z> \\ | \\ t^i\ R^y{}_t\ t^x \\ | \\ p,\ <t^i i^y s^z> \end{array} \qquad \begin{array}{c} [P]_1\, p\ <t^x i^y s^z> \\ | \\ t^i\ R^y{}_t\ t^x \\ | \\ p,\ <t^i i^y s^z> \end{array}$$

(where t^i is new to the branch), (for all times that are members of i^y)

$$\begin{array}{c} <P>_2\, p,\ <t^x i^y s^z> \\ | \\ i^j\ R^z{}_i\ i^y \\ | \\ p,\ <t^i i^j s^z> \end{array} \qquad \begin{array}{c} [P]_2\ p,\ <t^x i^y s^z> \\ | \\ i^j\ R^z{}_i\ i^y \\ | \\ p,\ <t^i i^j s^z> \end{array}$$

(where i^j is new to the branch and t^i is a member of it), (for all times of all intervals which access i^y that are members of s^z)

$$\begin{array}{c} <P>_3\ p,\ <t^x i^y s^z> \\ | \\ s^k\ R_s\ s^z \\ | \\ p,\ <t^i i^j s^k> \end{array} \qquad \begin{array}{c} [P]_3\, p,\ <t^x i^y s^z> \\ | \\ s^k\ R_s\ s^z \\ | \\ p,\ <t^i i^j s^k> \end{array}$$

(where s^k is new to the branch, i^j is a member of s^k, and t^i is a member of i^j), (for all times of all intervals of all spans which access s^z)

$$\begin{array}{c} <F>_1\, p,\ <t^x i^y s^z> \\ | \\ t^x\ R^y{}_t\ t^i \\ | \\ p,\ <t^i i^y s^z> \end{array} \qquad \begin{array}{c} [F]_1\, p\ <t^x i^y s^z> \\ | \\ t^x\ R^y{}_t\ t^i \\ | \\ p,\ <t^i i^y s^z> \end{array}$$

(where t^i is new to the branch), (for all times that are members of i^y)

$$\begin{array}{c} \langle F\rangle_2\, p, \langle t^x i^y s^z\rangle \\ | \\ i^y\ R^z{}_i\ i^j \\ | \\ p, \langle t^i i^j s^z\rangle \end{array} \qquad\qquad \begin{array}{c} [F]_2\ p, \langle t^x i^y s^z\rangle \\ | \\ i^y\ R^z{}_i\ i^j \\ | \\ p, \langle t^i i^j s^z\rangle \end{array}$$

(where i^j is new to the branch and t^i is a member of it), (for all times of all intervals which i^y accesses that are members of s^z)

$$\begin{array}{c} \langle F\rangle_3\ p, \langle t^x i^y s^z\rangle \\ | \\ s^z\ R_s\ s^k \\ | \\ p, \langle t^i i^j s^k\rangle \end{array} \qquad\qquad \begin{array}{c} [F]_3\ p, \langle t^x i^y s^z\rangle \\ | \\ s^z\ R_s\ s^k \\ | \\ p, \langle t^i i^j s^k\rangle \end{array}$$

(where s^k is new to the branch, i^j is a member of s^k, and t^i is a member of i^j), (for all times of all intervals of all spans which s^z accesses)

In addition, where n can be either1, 2 or 3, we have:

$$\begin{array}{c} \neg \langle P\rangle_n\ p, \langle t^x i^y s^z\rangle \\ | \\ [P]_n \neg\, p, \langle t^x i^y s^z\rangle \end{array} \qquad\qquad \begin{array}{c} \neg\, [P]_n\ p\ \langle t^x i^y s^z\rangle \\ | \\ \langle P\rangle_n \neg\, p, \langle t^x i^y s^z\rangle \end{array}$$

$$\begin{array}{c} \neg \langle F\rangle_n\ p, \langle t^x i^y s^z\rangle \\ | \\ [F]_n \neg\, p, \langle t^x i^y s^z\rangle \end{array} \qquad\qquad \begin{array}{c} \neg\, [F]_n\ p\ \langle t^x i^y s^z\rangle \\ | \\ \langle F\rangle_n \neg\, p, \langle t^x i^y s^z\rangle \end{array}$$

7.2.4. *Some Tense Tableaux*

1) First, I will show that $[F]_1 \neg p \nvdash [F]_2 \neg p$:

$[F]_1 \neg p, \langle t^1 i^1 s^1 \rangle$
$\neg\ [F]_2 \neg p, \langle t^1 i^1 s^1 \rangle$
|
$\langle F \rangle_2 \neg \neg p, \langle t^1 i^1 s^1 \rangle$
|
$i^1 R^1_i i^2$
|
$\neg \neg p, \langle t^2 i^2 s^1 \rangle$
|
$p \langle t^2 i^2 s^1 \rangle$

Here we've applied all the rules we can, and the tableau has not closed, so it's open. Thus, even if something will never happen at the first order, it can still be true that it will happen at the second order.

3) Next, I will show that $p \vdash \langle F \rangle_1 \langle P \rangle_1 p$:

$p, \langle t^1 i^1 s^1 \rangle$
$\neg \langle F \rangle_1 \langle P \rangle_1 p, \langle t^1 i^1 s^1 \rangle$
|
$[F]_1 \neg \langle P \rangle_1 p, \langle t^1 i^1 s^1 \rangle$
|
$t^1 R^1_t t^2$
|
$\neg \langle P \rangle_1 p, \langle t^2 i^1 s^1 \rangle$
|
$[P]_1 \neg p, \langle t^2 i^1 s^1 \rangle$
|
$\neg p, \langle t^1 i^1 s^1 \rangle$ (From the previous node and $t^1 R^1_t t^2$)
x

So if something is the case at the first order, it will at some time be the case at the first order that it was at some time the case at the first order.

3) Finally, I will show that $p \nvdash \langle F \rangle_2 \langle P \rangle_1 p$:

$$p, \langle t^1 i^1 s^1 \rangle$$

$$\neg \langle F \rangle_2 \langle P \rangle_1 p, \langle t^1 i^1 s^1 \rangle$$

|

$$[F]_2 \neg \langle P \rangle_1 p, \langle t^1 i^1 s^1 \rangle$$

|

$$i^1 R^1{}_i i^2$$

|

$$\neg \langle P \rangle_1 p, \langle t^2 i^2 s^1 \rangle$$

|

$$[P]_1 \neg p, \langle t^2 i^2 s^1 \rangle$$

|

$$t^3 R^2{}_t t^2$$

|

$\neg p, \langle t^3 i^2 s^1 \rangle$ (Note that t^3, despite its name, is *earlier* than t^2)

Here we've applied all the rules we can, and the tableau has not closed, so it's open. So if something is the case at the first order, it *does not* follow that it will at some time be the case at the second order that it was at some time the case at the first order. In other words, what is the case may, at some second-order-future time, *never* have been the case (at the first order).

7.2.5. *Changes in Time*

None of this is to say that I think this account is true of actual time. If it isn't, it can nevertheless be applied to the semantics of fiction. One can make a comparison with Graham Priest's story "Sylvan's Box" (Priest 1997), in which Priest argued that a contradiction can be true according to a story without everything being true according to it. Similarly, in many science fiction stories there are certain time travel scenarios in which something happens which seems to "change the past," but typically it is not said that there is any one time at which something both happens and fails to happen. HOTL could be used to reason about what is true according to such scenarios. If the time-traveler was "originally" not at some event, one can nevertheless suppose that there is a second-order-later interval at which they are, and that their presence results in different first-order moments being part of the first-order timeline of the second interval than those that were (at the second order) part of the first-order timeline of the first interval. This sort of change in first-order time over higher-order time is importantly different from a dialetheic view: HOTL does not require that there be any point of evaluation at which a contradiction is true—indeed, it forbids it. HOTL thus allows one to represent such a "change in time" without representing that there are any true contradictions.

So, if there are higher-order tenses, HOTL can be used to model certain structural properties of time; and if not, it can still be used to model the semantics of fiction. In any case I think our discussion shows HOTL to be interesting in its own right, and thus to be worth exploring.

7.3 *Laws of Nature*

There are some things about the universe that cry out for explanation. One is the fact that the universe is *orderly*. In a great many instances, similar causes produce similar effects, and the past can serve as a guide to the future. To make this point clearer and more precise, I think it will be instructive to consider the views of causation put forward by the philosophers Brand Blanshard[1] and A.C. Ewing[2], who gave similar arguments for the claim that causal relations are "logically necessary". Given that their views of logic are somewhat unorthodox by the standards of analytic philosophers, I think it would be more accurate and less confusing to talk of *metaphysical necessity* in causation, and I will do so in what follows.

A "rational reconstruction" of their arguments goes something like this: If causal connections are not metaphysically necessary, the fact that similar effects follow upon similar causes, or that there are certain, seemingly exceptionless regularities in nature—which can be

[1] *The Nature of Thought (second edition), vol. 2* , Ch. XXXII, "Concrete Necessity and Internal Relations"; *Reason and Analysis,* Ch. XI, "Necessity in Causation".
[2] *Non-Linguistic Philosophy*: Ch. VI, "Causation and Induction".

expressed in laws of nature—is quite remarkable. If “anything can cause anything”, as Humeans sometimes say, we have a tremendous coincidence, “an outrageous run of luck”, as Blanshard puts it[3], which is comparable to rolling a die and getting a 4 a trillion times in a row. But if causal connections *are* metaphysically necessary, we have a good explanation for the fact that similar effects follow upon similar causes, or that there are exceptionless regularities in nature: they obtain because they must. If events of type B *necessarily* follow upon events of type A, any token A event will be followed by a token B event. Granting that, we may be able to justify instances of inductive inference that fit the following schema: Events of type A *have always been* followed by events of type B, hence, events of type A *will always be* followed by events of type B.

The argument for this schema is this: In certain cases we take ourselves to have established that every observed event of type A has been followed by an observed event of type B. We also note that, since type A events are observed *very* frequently, it is implausible, though possible, that their association with type B events is a matter of chance. There appear to be two alternatives: Either the association is an a coincidence, or there is a necessary connection between them, albeit one that we may not be able to discern. Next we consider the principle of Inference to the Best Explanation (IBE): This principle says, very roughly, that if we have multiple hypotheses vying to account for some phenomenon, it is most reasonable to accept that the hypothesis which best explains it is the true one. If having *any* explanation is rationally preferable to having none—assuming we have no evidence which rules out all the candidate explanations, or which renders them improbable—IBE tells us that it is more reasonable to

[3] *The Nature of Thought, vol. 2*, p. 505 .

accept an explanatory hypothesis over a non-explanatory one. Since coincidence is no explanation, in the present case IBE counsels us to accept the hypothesis that there is a metaphysically necessary connection between type A events and type B events—as long as there is no other alternative. Because of this necessary connection, we can conclude that type A events will always be followed by type B events, just as they always have been.

Note that we have justified the schema neither deductively nor inductively: We have not deduced, and neither have we seen through "rational insight", that it is necessary that type A events will always be followed by type B events based on knowledge of their natures, nor have we concluded that type A events will always be followed by type B events just because they have always been so followed in the past. We have relied instead on IBE. Neither have we invoked the principle of sufficient reason or the idea that every event must have a cause; we have only said that it is *more reasonable* to believe in a necessary connection than an incredibly extensive coincidence. Thus the objections that can be raised against such principles cannot be raised against the present argument.

How, though, could there be a necessary connection of this sort? As Hume argued at length in the *Treatise* and first *Enquiry*, such connections cannot be logically necessary, and the most minute acquaintance with causes and their effects gives us no a priori insight as to why events of the one type should be followed by events of the other. However, if one accepts that there are different orders of metaphysical modality there is one explanation that naturally presents itself: Though these connections are *necessary*, they are *contingently* necessary, and *that* is why we have no a priori insight into them. Laws of nature may be conceived of as accidental regularities writ large: Just as it may be that some regularities hold throughout the universe as a matter of contingent fact, it may be that some regularities hold throughout the *actual galaxy* as a matter of contingent fact. This would be contingent, not because it is contingent what goes on within that galaxy—what occurs within any world is absolutely essential to the identity of that world, and which worlds are members of a given galaxy is absolutely essential to the identity of that galaxy—but rather because (a) it is contingent whether *that* galaxy is the actual galaxy, and (b) that regularity will not in general hold in other galaxies, since they may contain worlds not contained in the actual galaxy, and given that the regularity is not logically necessary it will fail to hold in some worlds.

Why assume that there are such regularities, which hold throughout vast swaths of logical space? The reason is simple: Since, for any worlds you please, there is some galaxy containing exactly those worlds, it follows that for any potential regularities you please, there will be some galaxy which contains only worlds where those regularities hold.

Some may question how much of an improvement this is over a more orthodox regularity view, according to which there is only one order of modality, and natural laws are merely accidental regularities. My response would be that one important difference between that view and the one I've presented is that even though laws of nature are *contingently* necessary on my view, they are still contingently *necessary*. That means that natural laws hold throughout every world in the actual galaxy, or at least throughout every world in the actual galaxy which this world accesses. This in turn means that what these laws prescribe is *counterfactually invariant* (at least at the first order of modality): If it is a law of nature that type A events will always be followed by type B events, then type A events would always have been followed by type B events *no matter what*. Granted, it is higher-order contingent that type A events would always have been followed by type B events no matter what: If the highest order at which that is necessary is the first order, then at every higher order it is a contingent fact that it is first-order necessary that type A events are always followed by type B events. Thus there is no explanation as to why that is first-order necessary. But as I see things, a *lack of explanation*, unlike necessity, is not "hereditary downwards". We are indeed supposing that there is no explanation as to why *it is first-order necessary that* A events are always followed by type B events, but even so, there *is* a perfectly good explanation as to why type A events are always followed by type B events: It is first-order-necessarily so.

8. *The Philosophy of Religion*

In this section we shall turn to the philosophy of religion. If there are higher-order modalities, there are higher-order contingencies; and if there is also an absolutely necessarily existent, omnipotent God, He ought to have control over them. A restricted version of Descartes' view that necessary truths were subject to God's will would be true. God would not, on this view, have the power to make contradictions true, or to make two plus two equal to five; but He could decide, for example, to make the thesis of the essentiality of origins true or false by deciding which galaxy to actualize.

If God did have that kind of control over the modal landscape, it would have implications for the problem of evil; in particular, for the *soteriological problem of evil.* This problem involves the issue of how, on the Christian view and others like it, the existence of a just and loving God is compatible with some people being condemned to hell. This is especially problematic for *Molinist* views, according to which God has *middle knowledge*—that God can know, independently of which possible world is actual, what agents would freely choose to do in certain circumstances, even though those agents possess libertarian free will. God can thus choose to actualize those circumstances and in a sense exercise a non-causal control over what non-deterministic agents *freely choose* to do. The question, then, is why God didn't choose to actualize circumstances in which agents freely act in such a way that no one gets condemned to hell.

One way of responding to this is to say that human nature is such that that is not possible, that some always freely choose to reject God. God simply can't actualize a world in which this isn't so. However, if my framework constitutes the correct view of modality, God ought to have control over what "human nature" is like: Even if some always freely choose to reject God in every world in the actual galaxy, God could have chosen to actualize another galaxy in which this isn't the case. Why, then, didn't God do so? One could respond that some freely choose to reject God not merely in every world in the actual galaxy, but in *every possible world there is*. But that would make the claim that some freely reject God co-extensive with a logical truth; and that, in my view, is an implausibly strong status for such a claim to have: If a claim doesn't hold as a matter of logic, there ought to be some logically possible world in which it is false. If my framework is correct, then, this Molinist defense against the soteriological problem of evil simply doesn't work.

9. *Conclusion*

In this paper I have examined the possibility and prospects of two novel logical frameworks, which I have called *higher-order modal logic* and *higher-order tense logic*. I have outlined their motivation, as well as their syntax and semantics. I have shown how to establish derivability claims about them via semantic tableaux, and have constructed a few of them myself. Finally, I have explored some of their implications for modal metalogic, the philosophy of language, metaphysics, and the philosophy of religion. It is my hope that the reader will agree with me that they are frameworks which are both intriguing in their own right and potentially fruitful, having interesting implications for our understanding of modality and a variety of other issues of philosophical significance.

Bibliography

Blanshard, Brand. *The Nature of Thought, 2nd edition, vol. two.* George Allen & Unwin Ltd., London, 1948

Blanshard, Brand. *Reason and Analysis*. Open Court, Chicago and La Salle, Illinois, 1964

Ewing, A. C. *Non-Linguistic Philosophy*. George Allen & Unwin Ltd., London, 1968

Hume, David. *An Enquiry concerning Human Understanding*, Edited by Tom L. Beauchamp. Oxford University Press, Oxford 1999.

Hume, David. *A Treatise of Human Nature*, Edited by David Fate Norton and Mary J. Norton. Oxford University Press, Oxford 2001.

Kripke, Saul A. *Naming and Necessity*. Cambridge, Massachusetts: Harvard University Press, 1980.

Lewis, David. *On the Plurality of Worlds.* Blackwell Publishing 1986, Malden Massachusetts.

Plantinga, Alvin. *The Nature of Necessity,* Oxford University Press 1974, Oxford.

Priest, Graham. *An Introduction to Non-Classical Logic: From If to Is, 2nd edition,* Cambridge University Press 2008, New York, New York.

Priest, Graham. “Sylvan's Box: A Short Story and Ten Morals”, *Notre Dame Journal of Formal Logic,* 38: 573–81, 1997.

Wittgenstein, Ludwig. *Tractatus Logico-Philosophicus.* Translated by C. K. Ogden. Barnes & Noble Books 2003; first published 1922.