A Primer on Logic

Part 1:

Preliminaries and Vocabulary

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1. *An Easy $10.00?*

Suppose someone were to bet you $10.00 that you would fail a seemingly simple test of your reasoning skills. Feeling confident in your abilities, you accept. The test works like this: There are four cards, numbered (i)—(iv). Each card has a letter on one side and a number on the other. Your goal is to check to see if the following claim is refuted by any of the cards. The claim is that, if a card has a vowel[1] on one side, then it has an odd number on the other. To win, you must turn over *all* of the cards you need to to check the claim, and *no more* than the cards you need to to check the claim. The cards are displayed like so:

a	3	c	2
(i)	(ii)	(iii)	(iv)

Which of the cards do you need to turn over to check to see if the claim holds? Stop reading this essay for a minute and try to figure it out. I don’t mind waiting.

[1] In case you’ve forgotten or are unsure which letters are vowels, they are *a*, *e*, *i*, *o* *u*, and (sometimes) *y* and *w*.

Okay, are you ready? The correct answer is that only cards (i) and (iv) should be turned over. If you're like most people, you got it wrong, in which case you would be out $10.00!

Card (i) should be turned over because it has the letter 'a', a vowel, showing. The claim you are testing says that, *if* a card has a vowel on one side, *then* it has an odd number on the other. Card (i) might have an odd number on the other side, in which case the claim has not been refuted. Then again, it might have an *even* number on the other side, in which case the claim *has* been refuted. Since you don't know whether the number on the other side of the card is odd or even, you need to turn it over. When presented with tests such as this, most people do indicate, correctly, that the card corresponding to this one should be turned over. For the other cards, the answer may not seem so obvious.

Card (ii), however, should *not* be turned over. Given the way the test is set up, there may be a vowel or a consonant on the other side of the card. If the letter is a vowel, the claim is not refuted, for then there is indeed a vowel on one side and the number 3, an odd number, on the other. If the letter on the other side is a consonant, the claim is also not refuted, for it can only be refuted if there is a *vowel* on one side and an even number on the other. If there is a *consonant* on one side, it doesn't matter whether the number on the other side is odd or even.

Card (iii) has the letter 'c'—a consonant—showing, and so it too should not be turned over for the reason just given: If a card has a consonant on one side, it doesn't matter whether the number on the other side is odd or even.

Finally, card (iv) should be turned over because it has a 2 showing, which is an even number. If the letter on the other side of the card is a consonant the claim is not refuted for a reason that should now be clear. But if the letter on the other side is a *vowel*, the claim *is* refuted, for it says that a card with a vowel on one side must have an *odd* number on the other.

From the foregoing, one can see why cards (i) and (iv), and no others, need to be turned over in order to test the claim.

The test to which you have just been subjected is called a *Wason selection task*. There are many different variants of it. When the claim being tested is relatively abstract, as it was here, people tend to do poorly. When the claim is more concrete—e.g., that if a person is under twenty one they can't legally drink beer—people tend to do better. However, logic applies to all subject matter, both abstract and concrete. I have chosen to use an abstract claim so you can see how difficult it can be to think logically.

2. *Arguments*

But *what is it* to think logically? By the time you've finished reading this essay I hope to have provided you with an answer to this question. We shall begin in Part 1 by examining the notion of an *argument*. An argument, as we will understand it, is not a heated dispute between two or more people. Neither is it a more civil exchange between people with opposing views, as political debates are ideally supposed to be. Instead, we will regard an argument as a sequence of *sentences*.

Not just any sentences will do, though. The sentences being considered here are *declarative* sentences, as opposed, for example, to *interrogative* sentences—questions—and *imperative* sentences—commands. For the sake of brevity we will henceforth call declarative sentences *statements*. Unlike questions and commands, statements say something about the world. That is, they represent things as being a certain way, and are true if things are that way and false if they are not. So if I say that a certain cat is laying on a certain mat, I have made a statement, and what I said is true if the cat is laying on the mat, and false if it is not. In this respect statements differ from questions and commands. Questions may have right or wrong answers, and commands may be obeyed or disobeyed, but they are certainly neither true nor false.

Arguments, then, are sequences of statements. They consist of a set of one or more *premises*, statements that are supposed to give support for a further statement which is called the *conclusion*. Consider the following argument, a variant of what is probably the oldest and most widely cited arguments in Western philosophy:

All humans are mortal.

Socrates is human.

Socrates is mortal.

In this argument, the first two statements are the premises and the last is the conclusion. The line serves the purpose of marking off the premises of the argument from the conclusion. The premises give support to the conclusion in the sense that they *entail* it. We say that the premises of an argument entail its conclusion when it is impossible for all the premises to be true and the conclusion to be false.[2]

You should be aware that the notion of entailment doesn't just apply to premises and conclusions. In general, it can apply to any statements or sets of statements, whether they can be thought of as being premises or conclusions or not. The only requirement is that one statement is entailed by another statement or set of statements in all and only those cases where it is impossible for the former statement to be false while the latter statement(s) are all true.

In our sample argument, it is impossible for all humans to be mortal and for Socrates to be human, and for it also to be the case that Socrates is *not* mortal. It is important to realize that when I say that *all* humans are mortal I do *not* mean that the vast majority of humans are mortal, while failing to mention certain exceptions because they are few and far between. I mean that *every single human* is mortal, *period*. That being so, if all humans are mortal and Socrates is human, Socrates *must* be mortal. Hence, if it turns out that Socrates is *not* mortal, it follows that either not all humans are mortal, or that Socrates is not human, or perhaps both. Arguments such as this, in which the premises entail the conclusion, are called *valid*. If, in addition, all of the

[2] There are other ways in which the premises of an argument can support its conclusion. In *inductive* arguments—as opposed to *deductive* arguments, which we are considering here—the premises may support the conclusion in the sense that their truth would *render it more probable* than it would otherwise be. We will say no more about inductive arguments in this essay.

premises of an argument are true, the argument is called *sound*. Since all sound arguments are valid, and since by definition all sound arguments have true premises, all sound arguments have true conclusions.

A word of caution is in order. Under no circumstances should you call an argument *true*. Besides the fact that philosophers and logicians never talk that way, there is the fact that when talking with someone your interlocutor may be unsure as to what exactly you mean by calling an argument *true*. You might mean that it is valid, or that it is also sound, or perhaps that its premises and conclusion are all true. Regarding this last possibility, it is important to note that an argument may have all true premises and a true conclusion while being invalid, unsound or both.[3] Consider the following argument:

Most basketball players are over five feet tall.

<u>The Moon orbits the Earth.</u>

Aristotle was a philosopher.

As things are, all three of these statements are true. Nevertheless, the premises of this argument do not entail the conclusion. Most basketball players could still have been over five feet tall, and the Moon could still have orbited the Earth, even if Aristotle had chosen to be a fisherman instead of a philosopher. Thus both premises of this argument could have been true even though its conclusion was false, making it invalid and unsound.

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[3] An argument that is unsound may still be valid, but one or more of its premises must be false—if they were all true, the argument would be sound after all. When an argument is both valid and unsound, its conclusion may or may not be true. An example of a valid but unsound argument is:

If the Earth is flat then the Earth doesn't have an equator.

<u>The Earth is flat.</u>

The Earth doesn't have an equator.

In this case the first premise is true, the second premise is false, and the conclusion is false.

3. *Consistency and Inconsistency*

I will now introduce two other logical notions, those of *consistency* and *inconsistency*. While they do not directly concern arguments, it is important for you to become acquainted with them. A set of statements is *consistent* if it is possible for all of the statements that compose it to be true together. Correspondingly, a set of statements is *inconsistent* if it is *not* possible for all of the statements that compose it to be true together. Alternatively, we can understand consistency and inconsistency in terms of entailment: A set of statements is *consistent* if no member of the set entails that any member of the set is false, otherwise the set is *inconsistent*.

To make these notions clear, consider the following set:

1. No man is both tall and fat.

2. Chris is tall.

3. Chris is fat.

4. Chris is a man.

It doesn't take much thought to realize that this set is inconsistent. If Chris is tall, fat, and a man, then some man is both tall and fat, which is precisely what statement (1) denies. So statements (1)—(4) cannot all be true, and are inconsistent by the first definition of inconsistency. The set consisting of (2)–(4) entails that (1) is false, and (1) entails that at least one of (2), (3) and (4) is false, although it is consistent with any one of them taken by itself, and also with any two of them taken together. So the set is also inconsistent by the second definition.

Now consider this set:

5. Jones is sitting and Jones is not sitting.

6. All whales are mammals.

7. Mt. Everest is the world's tallest mountain.

This set is also inconsistent by either definition of 'inconsistent'. The reason is that (5) cannot be true—provided we understand it as saying that Jones is both sitting and not sitting *at the same time*. Granting that, since (5) cannot be true by itself, it cannot be true together with (6) and (7) either. Furthermore, because (5) cannot be true, every statement entails that (5) is false, including (5) itself![4] We may say that (5) is *self-inconsistent*. We may also say that self-inconsistent statements are *infectious*, because their inconsistency spreads: Any set of statements that contains a self-inconsistent statement is also inconsistent, even if all the other statements in the set are consistent with themselves and each other.

4. *Conclusion*

Now that we have some understanding of some basic logical notions we can move on. In Part 2 I'll discuss *argument schemas*, abstract "forms" or patterns that different arguments have in common. We'll learn how to tell which schemas are valid and why.

[4] Recall that one statement entails another when it is impossible for the former to be true while the latter is false. Since (5) cannot be true, it is necessarily false. So the statement "(5) is false" is necessarily true, and no statement can be true while *it* is false, for it is necessarily true. Thus every statement entails the statement "(5) is false", and by extension every statement entails the falsity of (5).